A Fermion Doublet With Chiral Gauge Interaction On A Lattice

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Abstract

We present a new staggered discretization of the Dirac operator. Doubling gives only a doublet of Dirac fermions which we propose to interpret as a physical (lepton or quark) doublet. If coupled with gauge fields, an $(1+\gamma^5)$ chiral interaction appears in a natural way. We define a generalization for curved background which does not require tetrad variables. The approach suggests a natural explanation for the three fermion families.

1 Introduction

In this paper we present a new discretization of the Dirac equation. In comparison with staggered fermions it creates not four but only two flavours of Dirac fermions. This has been reached by placing not only different spin components, but also their real and imaginary parts into different nodes. These sixteen steps of freedom (two fermions) can be understood, in some sense, as the result of "doubling" of a real scalar step of freedom $\varphi(n)$ on the lattice.

Moreover, these two fermions live on different sub-lattices (ψ_o on "odd" nodes, ψ_e on "even" nodes), thus, we obtain also a single fermion (eight steps of freedom) on the lattice by omitting half of the lattice nodes. But, if the neutrino is a Dirac particle, all fermions appear in doublets of Dirac particles. In this context, the appearance of a fermion doublet in this discretization may be not a bug but a feature.

In our approach the complex structure is an operator among others. Moreover, there is no natural complex structure, but, instead, a quaternionic

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structure. The choice of a complex structure corresponds to a choice of a direction. To avoid unnecessary symmetry breaking we propose to consider a vector field instead of a scalar field on the lattice. In this case, each component has its natural preferred direction and therefore a natural complex structure. This gives in a natural way three generations of our doublet.

Once a complex structure is defined, we can define the operator γ^5, which becomes a shift in the preferred direction. It exactly anticommutes with D. On the other hand, $(\gamma^5)^2 \neq 1$ but a shift by $2a$ in the preferred direction. The subspaces $\gamma^5 = \pm 1$ have an uncertain definition but a clear physical interpretation: they define the subspaces $\psi_o \approx \pm\psi_e$. An $1 + \gamma^5$ type chiral interaction with a gauge field appears in a straightforward way from the typical lattice gauge term $A_{n,n+a_i}(\phi(n) + \phi(n + a_i))$: If $\gamma^5 \approx 1$ it becomes $\approx A_i\phi(n)$, but if $\gamma^5 \approx -1$ it becomes almost zero.

The discretization may be generalized to a curved metric background. The fermion doublet may be, in this context, described without any tetrad variables by the sixteen-dimensional de Rham complex of four-dimensional differential forms $\Lambda = \sum_{k=0}^{n} \Lambda^k$. On the space of these differential forms exists a natural "Dirac operator" (in the sense of a square root of the harmonic operator) as well as a natural discretization for a general mesh.

In appendix A we consider the connection of this concept with a generalization of Lorentz ether theory to gravity. In this context, the proposal to explain fermion families as natural steps of freedom for ether crystal dislocations becomes really meaningful. Moreover, we define a natural $SO(3)$ gauge field which measures mesh irregularity which we propose as a candidate for the $SU(2)$ part of electroweak theory.

2 A Real Representation Of The Dirac Algebra

We forget – for some time – about the complex structure. Instead of the usual representations with four complex fields, we use the following eight-dimensional real representation:

$$\gamma^0\partial_0 - \gamma^i\partial_i = \begin{pmatrix} \partial_0 & \partial_1 & \partial_2 & & \partial_3 & & & \\ -\partial_1 & -\partial_0 & & \partial_2 & & \partial_3 & & \\ -\partial_2 & & -\partial_0 & -\partial_1 & & & \partial_3 & \\ & -\partial_2 & \partial_1 & \partial_0 & & & & \partial_3 \\ -\partial_3 & & & & -\partial_0 & -\partial_1 & -\partial_2 & \\ & -\partial_3 & & & \partial_1 & \partial_0 & & -\partial_2 \\ & & -\partial_3 & & \partial_2 & & \partial_0 & \partial_1 \\ & & & -\partial_3 & & \partial_2 & -\partial_1 & -\partial_0 \end{pmatrix} \tag{1}$$

In the context of this representation, it seems also natural to define the following operators β^i:

$$m_i\beta^i = \begin{pmatrix} & m_1 & m_2 & & m_3 & & & \\ m_1 & & & m_2 & & m_3 & & \\ m_2 & & & -m_1 & & & m_3 & \\ & m_2 & -m_1 & & & & & m_3 \\ m_3 & & & & & -m_1 & -m_2 & \\ & m_3 & & & -m_1 & & & -m_2 \\ & & m_3 & & -m_2 & & & m_1 \\ & & & m_3 & & -m_2 & m_1 & \end{pmatrix} \tag{2}$$

The following operator equation holds:

$$(\gamma^0\partial_0 - \gamma^i\partial_i + m_i\beta^i)^2 = -\Box + m_i^2 \tag{3}$$

This can be easily seen – this operator iterates three times, in each coordinate direction, the same trick:[1]

$$\begin{pmatrix} A & (m_i + \partial_i)I \\ (m_i - \partial_i)I & -A \end{pmatrix}^2 = (A^2 + (m_i + \partial_i)(m_i - \partial_i)I) \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \tag{4}$$

As follows immediately, the γ^μ define a representation of the Dirac matrices, and the matrices β^i fulfil the following anticommutation relations:

$$\beta^i\beta^j + \beta^j\beta^i = \delta^{ij} \tag{5}$$

[1]This observation also suggests how to iterate this construction to arbitrary dimension.

and anticommute with all γ^μ:

$$\beta^i\gamma^\mu + \gamma^\mu\beta^i = 0 \tag{6}$$

It is also easy to see (and to generalize to arbitrary dimension) that

$$\gamma^0(\gamma^1\beta^1)(\gamma^2\beta^2)(\gamma^3\beta^3) = 1. \tag{7}$$

The operator $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$ depends on the complex structure. The obvious replacement which does not depend on it we denote with *iota*:

$$\iota = \gamma^0\gamma^1\gamma^2\gamma^3 = \beta^1\beta^2\beta^3 \qquad \iota\gamma^\mu + \gamma^\mu\iota = 0 \qquad (\iota)^2 = -1 \tag{8}$$

3 Discretization Of The Dirac Equation

This representation is appropriate for a discretization of the Dirac equation on a regular hyper-cubic lattice. It can be obtained in a quite simple way: We start with a naive central difference approximation

$$\partial_i\psi(n) \to \frac{1}{2a_i}(\psi(n+a_i) - \psi(n-a_i)). \tag{9}$$

We obtain the usual $2^4 = 16$ doublers. Fortunately, eight pairs of doublers decouple in a really simple way: It is sufficient to hold only one of the eight real components ψ^a per node. On the three-dimensional reference cube $(\varepsilon_1, \varepsilon_2\varepsilon_3), \varepsilon_i \in \{0, 1\}$ we obtain the following locations for the eight components:

$$\begin{array}{ll} \psi^0 \text{ located at } (0,0,0), & \psi^4 \text{ located at } (0,0,1); \\ \psi^1 \text{ located at } (1,0,0), & \psi^5 \text{ located at } (1,0,1); \\ \psi^2 \text{ located at } (0,1,0), & \psi^6 \text{ located at } (0,1,1); \\ \psi^3 \text{ located at } (1,1,0), & \psi^7 \text{ located at } (1,1,1); \end{array} \tag{10}$$

What remains are sixteen steps of freedom (eight steps of freedom on two time steps which we need because of our use of central differences in time) which corresponds to a doublet of Dirac fermions[2]. Note that our discretization may be interpreted as a way to discretize the d'Alembert equation

[2]In comparison, the standard staggered fermions approach reduces the doublers only by a factor four. Because we ignore the complex structure of the standard representation, we are free to place "real" and "imaginary" part of the complex fields into different nodes. This gives the additional reduction by factor two.

for a single scalar step of freedom $\varphi(n)$ with central differences, which gives $2^4 = 16$ doublers.

Now, the last doublet decouples too, and only in a slightly less trivial way: We can distinguish "even" and "odd" nodes on the full space-time lattice. The central difference equations on even (odd) nodes connects only values on odd (even) nodes. Thus, we obtain two fermions ψ_e and ψ_o on even resp. odd nodes. On the four-dimensional reference cube $(\varepsilon_0, \varepsilon_1, \varepsilon_2\varepsilon_3), \varepsilon_i \in \{0, 1\}$ we have

$$\begin{array}{ll} {\psi_e}^0 \text{ located at } (0,0,0,0), & {\psi_e}^4 \text{ located at } (1,0,0,1); \\ {\psi_e}^1 \text{ located at } (1,1,0,0), & {\psi_e}^5 \text{ located at } (0,1,0,1); \\ {\psi_e}^2 \text{ located at } (1,0,1,0), & {\psi_e}^6 \text{ located at } (0,0,1,1); \\ {\psi_e}^3 \text{ located at } (0,1,1,0), & {\psi_e}^7 \text{ located at } (1,1,1,1); \end{array} \tag{11}$$

$$\begin{array}{ll} {\psi_o}^0 \text{ located at } (1,0,0,0), & {\psi_o}^4 \text{ located at } (0,0,0,1); \\ {\psi_o}^1 \text{ located at } (0,1,0,0), & {\psi_o}^5 \text{ located at } (1,1,0,1); \\ {\psi_o}^2 \text{ located at } (0,0,1,0), & {\psi_o}^6 \text{ located at } (1,0,1,1); \\ {\psi_o}^3 \text{ located at } (1,1,1,0), & {\psi_o}^7 \text{ located at } (0,1,1,1); \end{array} \tag{12}$$

But, instead of removing one sub-mesh, we propose to accept above doublers as a way to describe a physically meaningful flavour doublet. If the neutrino is a standard Dirac particle, then all fermions of the standard model appear in doublets.

4 Complex Structures

To understand the connection of this approach to usual fermions we need a complex structure – a multiplication with i. In our real representation this is an operator. It's basic property is $i^2 = -1$. To be compatible in the usual way with the Dirac equation, we also need $[\gamma^\mu, i] = 0$. This does not define the complex structure uniquely. We have three interesting candidates for a complex structure:

$$i = \beta^1\beta^2 = \iota\beta^3 \tag{13}$$

$$j = \beta^2\beta^3 = \iota\beta^1 \tag{14}$$

$$k = \beta^3\beta^1 = \iota\beta^2 \tag{15}$$

which together define a quaternionic structure:[3]

[3]The classical representation $ij = k$ can be obtained using reverse signs for i, j, k, but we prefer this sign convention because it gives $\gamma^5 = \beta^3$.

$$ij = -ji = -k;\ jk = -kj = -i;\ ki = -ik = -j;\ i^2 = j^2 = k^2 = -1 \quad (16)$$

For each candidate for a complex structure, we obtain an own operator $\gamma^5 =_{def} -i\iota$. For $i = \iota\beta^3$ we obtain $\gamma^5 = -\iota\beta^3\iota = \beta^3$.

4.1 Fermion Families and Lattice Distortions

Thus, it seems that to fix the complex structure we somehow have to break spatial symmetry. But there is a simple way out of this. Instead of one scalar step of freedom $\varphi(n)$ on the lattice, we can consider a vector field – thus, three components $\varphi^i(n)$. Now, each component has a natural "preferred direction" and, therefore, a natural complex structure.

Moreover, a vector on a lattice is a quite natural step of freedom. It is, for example, the natural way to describe lattice distortions with a shift vector field $u^i(n)$.

On the other hand, in the standard model we have three families – three copies of each fermion. This suggests to explain on the kinematic level the three fermion families using the hypothesis that the fundamental steps of freedom of the "theory of everything" are three-dimensional vector fields $u^i(n)$. This idea becomes really meaningful only in an ether-theoretical context, where relativistic symmetry is considered as not fundamental but an effective large scale symmetry, and distortions of an "ether crystal" are natural steps of freedom. We consider such an approach in more detail in appendix A.

For our symmetry problem – the choice of the complex structure – other solutions are possible. We can use them all in expressions like

$$iA^1 + jA^2 + kA^3 \quad (17)$$

which allows to connect with an $SU(2)$ or $SO(3)$ gauge field. We can also try to connect colors with spatial directions to define the preferred complex structure differently for each colored quark. Last not least, it may be really broken (possibly the $U(1)$ part of electroweak interaction). These different solutions do not contradict each other. But that the choice of the complex structure is not natural but requires a symmetry breaking of the quaternionic structure of spinors is a point which may be important for the understanding of the standard model.

5 Chiral Symmetry

Now we have to define the operator $\gamma^5 = \beta^3$ on the lattice. Note that it cannot be anymore a pointwise operator as for Wilson fermions and staggered fermions – it connects components which are located in different points. Thus, the original, continuous definition is not enough to define γ^5 on the lattice. But there is one very natural choice: the shift operator in z-direction. Modulo 2, on the reference cube, it is indeed γ^5. And, most important, it anticommutes with the Dirac operator even on the lattice:

$$\gamma^5 D + D\gamma^5 = 0 \tag{18}$$

But, different from the γ^5 in continuous theory, it's square is no longer 1, but a shift in z-direction. But this is a very nice and beautiful way to break exact chiral symmetry with $(\gamma^5)^2 = 1$. Note that γ^5 exchanges the even and odd fermion:

$$\gamma^5 \psi_e(n) = \psi_o(n - a_z) \qquad \gamma^5 \psi_o(n) = \psi_e(n - a_z) \tag{19}$$

This gives the subspaces $\psi_\pm = \frac{1}{2}(1 \pm \gamma^5)\psi$ a quite obvious physical meaning: they are the subspaces defined by $\psi_e \approx \pm\psi_o$. On the other hand, because ψ_e and ψ_o are located on different sub-meshs, we do not have a natural symmetric and exact definition of the projectors.

But the most beautiful surprise we find thinking about the connection to gauge fields. As usual, we describe gauge fields as located on edges. Now, a quite natural, symmetric lattice interpolation for interaction terms $A\psi$ between fermions and the gauge field is

$$A_{n+a_i,n} \frac{\psi(n + a_i) + \psi(n)}{2} \tag{20}$$

Thus, we obtain a dependency on terms of type $\psi(n + a_i) + \psi(n)$ in a natural way. Now, these terms are $\approx \psi(n)$ for ψ_+ but ≈ 0 for ψ_- – thus, our interaction terms with gauge fields have exactly the $(1 + \gamma^5)$ form of interaction we need in chiral gauge theory! Thus, in this discretization we have not only a possibility to describe chiral gauge fields, but they appear almost by themself, in a quite natural way.

This effect disappears if we have several components and the gauge field connects them. Thus, our $(1 + \gamma^5)$ type interaction works only inside the doublet. Outside the doublets we obtain a vector-like interaction – as it should be in chromodynamics.

6 The Dirac Operator On The De Rham Complex

The discretization may be generalized to the case of a general metric background. To be accurate, this is not really a generalization: The discretization considered above has been found as a specialization of this general scheme – in the attempt to understand if the differential-geometric Dirac operator described below has something to do with usual Dirac fermions.

6.1 Hodge Theory

The de Rham complex $\Lambda = \sum_{k=0}^{n} \Lambda^k$ consists skew-symmetric tensor fields of type $(0,k), 0 \leq k \leq n$ which are usually written as differential forms

$$\psi = \psi_{i_1 \dots i_k} dx^{i_1} \cdots dx^{i_k} \in \Lambda^k \tag{21}$$

The de Rham complex Λ has dimension 2^n in the n-dimensional space. The most important operation on Λ is the external derivative $d : \Lambda^k \to \Lambda^{k+1}$ defined by

$$(d\psi)_{i_1 \dots i_{k+1}} = \sum_{q=1}^{k+1} \frac{\partial}{\partial x^{i_q}} (-1)^q \psi_{i_1 \dots \hat{i}_q \dots i_{k+1}} \tag{22}$$

where $\hat{i}_q$ denotes that the index i_q has been omitted. It's main property is $\partial^2 = 0$. In the presence of a metric, we have also the important $*$-operator:

$$(*\psi)_{i_{k+1} \dots i_n} = \frac{1}{k!} \varepsilon_{i_1 \dots i_n} g^{i_1 j_1} \cdots g^{i_k j_k} \psi_{j_1 \dots j_k} \tag{23}$$

with $*^2 = (-1)^{k(n-k)} \mathrm{sgn}(g)$. The graduation is also important: $\varepsilon\psi = (-1)^k \psi$ if $\psi \in \Lambda^k$. The subspaces $\varepsilon = 1$ and $\varepsilon = -1$ have equal dimension 2^{n-1}. In this general context we can define the Laplace operator as

$$\Delta = \varepsilon(*^{-1} d * d - d *^{-1} d*). \tag{24}$$

Then, the Dirac operator (as it's square root) can be defined as

$$D = d + \varepsilon *^{-1} d * . \tag{25}$$

Indeed, we have $d^2 = 0$ and $d\varepsilon = -\varepsilon d$ because d changes the dimension by one and therefore the sign ε. Thus,

$$(d + \varepsilon *^{-1} d*)(d + \varepsilon *^{-1} d*) = \varepsilon *^{-1} d * d + d\varepsilon *^{-1} d* = \Delta. \qquad (26)$$

6.2 Discretization On The De Rham Complex

If we want to consider the approximation of some continuous object on a general mesh, the de Rham complex is a very natural object. Indeed, k-forms may be integrated over k-dimensional surfaces. Thus, if a general mesh (a cell complex) is given, a k-form defines a function on the k-dimensional cells of the mesh in a very natural way. For each cell we can define it's "characteristic form" $\chi^i(c_j) = \delta^i_j$ and decompose the general form on the mesh as $f(c) = f_i \chi^i(c)$. The external derivative defines in a similar natural way a derivative for functions on the mesh, with the same most important exact property $\partial^2 = 0$.

For the definition of the $*$-operator we need a metric and a dual mesh. The metric defines in a natural way for every cell c_i it's area $a_i = a(c_i) > 0$. In the Euclidean case and a triangulation, these values depend on each other, but in the general case they may be considered as independent variables which define the metric. In the following we consider them as given and defining the metric.

The dual mesh is a quite natural mesh which can be defined for every mesh: for each k-dimensional cell c_i of the original mesh it has a $(n-k)$-dimensional dual cell $\hat{c}_i$ which intersects the original cell in a single point orthogonally and with positive intersection index. Now, the metric defines as well a scalar value $\hat{a}_i$ for each cell of the dual mesh $\hat{c}_i$ – the area of the dual cell. Now, the $*$-operator on the mesh may be defined as

$$(*f)(c) = \frac{\hat{a}_i}{a_i} f_i \chi^i(c) \qquad (27)$$

Note that the dual of the dual mesh has the same cells, but in some dimensions with different orientations. Therefore, for $*^2$ there appears the factor $(-1)^{k(n-k)}$ as in the continuous case.

Note also that the discrete de Rham complex is connected in a simple way with a natural "regular refinement" step of a mesh: To refine a mesh in a regular way, we can put one node in the center of each cell, and connect nodes of neighbour dimension cells with edges if one of the cell is on the boundary of the other. This leads to a mesh where each cell has the topological structure of a unit cube.

For a simple cubic lattice on a flat background, this discretization scheme reduces to our proposal above.

7 Conclusion

The approach described here seem to solve some outstanding problems of chiral gauge theory. First, there is the general problem how to formulate a chiral gauge theory on the lattice, known as the "regularization problem": any consistent regularization that preserves the gauge symmetry must refer to the fermion representation.[4]. In our case, we describe chiral gauge theory only in a very special, two-dimensional representation, therefore it is not in conflict with this requirement. And it gives the representation we need in the standard model. Moreover, if neutrinos are Dirac particles, we don't need any other representations – only their combinations (one lepton and three quark doublets). Of course, a lot of (hopefully only technical) things have to be done to obtain a complete quantum chiral gauge theory in this way.

Second, it solves the fermion doubling problem [3] – it reduces the number of "doubled flavours" from four in the staggered fermion approach (which are unphysical) to two which already allow a meaningful physical interpretation as lepton or quark doublets.

We also obtain an interesting representation of fermions on a general metric background which does not need tetrad or triad variables for the gravitational field. Instead, the gravitational field may be described as usual by the metric tensor $g_{\mu\nu}$.

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A Fermions In The General Lorentz Ether

The generalization of the Lorentz ether to gravity proposed in [5] combines the ADM decomposition with harmonic gauge to obtain a classical condensed matter ("ether") interpretation: Once we fix a time coordinate, the four-dimensional metric decomposes into a positive scalar $\rho = g^{00}\sqrt{-g} > 0$, a shift vector field $v^i = g^{0i}/g^{00}$, and a three-dimensional Riemann metric defined by $p^{ij}\sqrt{p} = -g^{ij}\sqrt{g}$. The harmonic condition $\partial_\mu g^{\mu\nu}\sqrt{-g} = 0$ is then interpreted in terms of condensed matter physics as continuity and Euler equation:

$$\partial_t \rho + \partial_i(\rho v^i) = 0 \quad (28)$$
$$\partial_t(\rho v^j) + \partial_i(\rho v^i v^j + p^{ij}) = 0. \quad (29)$$

Adding a non-covariant term to the GR Lagrangian to describe the Newtonian background

$$L = L_{GR} + L_{matter}(g_{\mu\nu}, \varphi^m) - (8\pi G)^{-1}(\Upsilon g^{00} - \Xi \delta_{ij} g^{ij})\sqrt{-g}, \quad (30)$$

we can obtain the harmonic condition as the energy-momentum conservation law. This scheme works in the other direction too: Assuming that the conservation laws of classical condensed matter theory appear via a special variant of Noether's theorem as energy-momentum conservation laws, it is possible to derive this Lagrangian as the most general Lagrangian. All material properties of the Lorentz ether become matter steps of freedom described by the matter Lagrangian. Especially the covariant character of its matter part, that means the Einstein equivalence principle, follows.

It has been noted [2] that this derivation does not extend (at least not in any obvious way) to tetrad or triad variables. Because the standard way to describe fermions on a curved background uses tetrad (or, in the context of an ADM decomposition, triad) variables, this makes the incorporation of fermions into this theory problematic. One way to solve this problem is, obviously, to find a way to describe fermions without tetrad or triad variables, what has been done in our approach.

In this context, it is important to consider the compatibility of our approach with the ADM decomposition. Using "comoving" spatial coordinates (which remain constant along the velocity field v^i), the harmonic operator of the metric $g_{\mu\nu}$ reduces to

$$\Box\psi = -(\rho\partial_t^2 - \Delta)\psi \quad (31)$$

where Δ is the standard three-dimensional (harmonic) Laplace operator of the metric p_{ij}. Thus, we ADM decomposition allows to reduce the harmonic equation to a "three-and-one-half-dimensional" equation

$$\sqrt{\rho}\partial_t\psi = \sqrt{\Delta}\psi. \quad (32)$$

Now it seems natural to use a mesh which is compatible with the ADM decomposition. We start with an arbitrary mesh on the spatial slice at a given

time t_0. On this three-dimensional mesh we can define the Dirac operator following the scheme described above. Then we can extend this mesh into a four-dimensional mesh. First, we extend the mesh nodes at t_0 following the comoving spatial coordinates into comoving world-lines. For the preferred time we use a global, regular lattice and simple central differences. Such a choice of a lattice and discretization is in natural agreement with a crystallic ether model.

A.1 A Natural $SO(3)$ Gauge Field For A General Three-Dimensional Mesh

Another reason why we think that the consideration of three dimensions will be helpful is that a general three-dimensional mesh defines in a simple and natural way an $SO(3)$ gauge field. Because at the Lie algebra level $so(3) = su(2)$ this may be a candidate for the $su(2)$ gauge field of the electroweak interaction.

To understand this gauge connection, remember that to define a gauge connection it is sufficient to define its integral over closed loops. Thus, let's define the integral over a closed loop starting from a general point (that means, inside a cell) and of general type (thus, the loop intersects only planar cells of the mesh).

Now, assume also that we have made a simple regular refinement step, therefore, the mesh is a cubic mesh. For each cubic cell, we fix some standard map into the regular cube. Then we start with a standard reper (as it comes from the standard cube) and use trivial parallel transport of the reper (as defined on the standard cube) inside the cells up to the boundary. On the boundary, one vector of the reper is always orthogonal to the boundary. If we come back to the original node, the transferred reper is also, by construction, a reper in this cube, its directions are parallel to the original reper. Nonetheless, it is in general a rotated reper. This rotation defines the gauge connection. It measures the irregularity of the mesh: For a regular cubic mesh it is trivial.

Now, the existence of a gauge field which is in such a natural way connected with mesh irregularity does not look like an accident of nature. Our hypothesis is that it is (at least connected in some way with) the $SU(2)$ gauge field of the electroweak interaction.

A.2 Conclusion

The approach presented here solves the problem how to describe fermions in the general Lorentz ether. It should be noted that the regularization problem

of chiral gauge theory becomes very important in ether theory if we want to use an "atomic ether" concept to regularize the theory. Indeed, in this case we need some lattice discretization for all parts of the standard model [1]. An "atomic ether" regularization, which interprets the density ρ as a particle density and velocity v^i as their average velocity is a quite natural way to quantize an ether theory. Thus, the solution of the lattice regularization problem for $1 + \gamma^5$ chiral interactions removes a serious obstacle on the way to an ether-based theory of quantum gravity.

Moreover, the ether approach allows to explain some important features of the standard model. Even the most trivial ether crystal gives three generations of a doublet of Dirac fermions. Crystal distortions are the most natural type of steps of freedom of an ether, which explains the three fermion families in a natural way. An ether crystal containing one type of "leptonic atoms" and three colored types of "quark atoms" already gives the whole fermionic content of the standard model. We obtain in a natural way chiral gauge interactions inside the doublets as well as vector gauge interactions between the doublets (chromodynamics). Moreover, there seems to be a nice, natural candidate for the $su(2)$ gauge field of electroweak theory. All this seems to be a good starting point for an ether-based understanding of the whole standard model and its unification with gravity.

References

[1] A. Bergman, personal communication

[2] L. Bourhis, personal communication

[3] R. Gupta, hep-lat/9807028

[4] M. Lüscher, hep-th/0102028

[5] I. Schmelzer, gr-qc/0104013